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AUTHOR Hamrin, Jeannie M.
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ABSTRACT

A computer model of memory (based on input, retrieval, and reaction time) was applied to the rapid processing of simple arithmetic facts (addition, subtraction, and multiplication) by 18 educable retarded adolescents when compared with 18 nonretarded fourth graders (of equal mental age) and 18 normal adolescents. Results for addition indicated that all three groups use a digital (rather than analog) processing model with the faster reaction time of the normal adolescents attributable not to memorization but faster perceptual and motor processes. Results for subtraction supported the digital model for the retarded and young Ss but suggested that the normal adolescents used a memory retrieval system. Results for multiplication also supported the digital model for all three groups indicating that addition and multiplication are based on the same common mental processing mechanism. Results support educational practices of encouraging retarded children to manipulate objects and actively participate as they learn arithmetic facts. (Appended are a list of terms with definitions and statistical tables.) (DB)

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# INFORMATION PROCESSING MODELS FOR ADDITION, SUBTRACTION, AND MULTIPLICATION IN THE MENTALLY RETARDED AND NON-RETARDED

by

Jeannie M. Hamrin

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# INFORMATION-PROCESSING MODELS FOR ADDITION, SUBTRACTION, AND MULTIPLICATION IN THE MENTALLY RETARDED AND NON-RETARDED

A paper presented at American Educational Research Association Annual Meeting, Toronto, Canada, March 29, 1978

Jeannie M. Hamrin

University of Maine at Portland-Gorham

## Introduction

Many investigators have hypothesized about the nature of memory and its mechanisms and laws. The computer has recently provided us with a useful model for memory which shifts the emphasis from the storage of memories to the processes of "input" and "retrieval". This phenomenon is investigated with regard to the rapid processing of simple arithmetic facts by utilizing Sternberg's (1967) reaction time (RT) paradigm.

The questions posed by this study were:

1. How does the distribution of computational processes of a sample of educable mentally retarded adolescents (MRs) compare with the distributions of non-retarded samples of fourth graders (equal-MAs) and adolescents (equal-CAs)?

2. After the most common model for each process has been determined, how do the groups differ in terms of input and output, retrieval processes, and errors?

The design of the study yielded over 40,000 pieces of data. Each of the 54 Ss was individually tested on 4 sets of 66 addition facts ($0 \leq p + q \leq 10$), 4 sets of 66 subtraction facts ($0 \leq p - q \leq 10$), and 6 sets of 36 multiplication facts ($0 \leq p \times q \leq 25$).

There were 54 Ss, nine males and nine females in each of three groups. The MR sample consisted of 18 non-institutionalized adolescents enrolled in a special education class ($\overline{CA}$=16.78, SD=1.78; $\overline{MA}$=8.45, SD=3.44; $\overline{IQ}$=63.47, SD=10.32).[1] Mentally retarded Ss were of both organic and

[1] IQ data is missing for two Ss. Fourteen scores are from the Stanford-Binet, Form L-M administered during the spring of 1972. Two scores are WISC.

familial etiology. Non-retarded Ss consisted of 18 fourth graders (CA=9.72, SD=0.45; MA=10.1; IQ=104.77, SD=7.56) and 18 volunteer high school students (CA=16.67, SD=1.07; IQ=114.86, SD=11.40).

The problems were visually presented to Ss by a slide projector and Ss responded verbally into a microphone. RTs were registered in milliseconds by an electronic timer receiving signals from the slide projector and voice-activated relay. The processing of the data from these experiments consisted of three phases; a data verification and reduction phase, a modelling phase consisting of linear regression analyses to estimate the model parameters, intercept ($\alpha$) and slope ($\beta$); and a statistical analysis phase consisting of analyses of variance to explore the differences in the processes of each group. These phases will be described in detail in the following sections and illustrated with sample outputs from the programs.

## Data Verification and Reduction

The RT MATRICES, the PROBLEM MATRICES and the ORDER VECTORS were inputted to the computer. (See Appendix for definitions). Problems, for which the subject gave erroneous answers or for which the RT was lost due to equipment failure or experimenter error, are defined as missing data, and entered as zero RT.

Between the RT MATRIX, the PROBLEM MATRIX and the ORDER VECTOR, the reaction time pattern for a particular process and subject is fully defined.

The program designed for the verification and reduction of the 'raw data' is called DATARED. Input to this program is the RT matrix, the PROBLEM matrix and the ORDER VECTOR. The DATARED program has the following input/output options:

Printing of the PROBLEM MATRIX for documentation and verification of the number of times each problem is represented. (See Appendix B, Exhibit 1).

Printing of the RT MATRIX with missing data flagged. (See Appendix B, Exhibit 2).

Printing of a RT MATRIX with times corrected for the experimental time constant and the "outliers" detected and flagged. This option also prints out the median and mean success latency as well as the standard deviation and number of errors for each problem. On this print-out the problems are shown in problem order and the slide series in the order they were shown to the subject. (See Appendix B, Exhibit 3).

Selection of the outlier limit, that is the value above which a RT is considered an outlier.

Selection of which slide series to be included or excluded. This option facilitates analysis of a possible learning effect.

Selection of digit combinations or problems to be included.

Modelling

This phase, the investigative part of the experiment, is characterized by a low volume of variable input, a large volume of computations, and a small volume of printed output. All the data processing for this phase was therefore done exclusively on an interactive terminal facility, CALL-OS.

The linear regression program REGRE out of the System/360 Scientific Subroutine Package was converted to run under CALL-OS. The data input routine was programmed to generate the model values for a MSL from the corresponding digit combinations. For instance, assume that we are investigating the addition process using the six models discussed in the paper. The value stored in the 5th row and the 3rd column of a subjects' MSL matrix is the MSL for the 4 + 2 problem. The data input routine will generate the values for the structural variable x for models 1 thru 6 (6,4,2,4,2 and 1.2041) and pass those values as independent variables to the REGRE program together with the MSL value as the dependent variable. When the whole MSL matrix has been processed in this way, a linear regression of MSL is performed over each of the independent variables, the values of model 1 thru 6, yielding $\alpha$, $\beta$, r and F values for each model and subject.

For each process, subject and problem selection, the model producing the largest F-value (Fmax) was considered the best descriptor of that subject's algorithmic process. By studying the various models and problem selections, the best fitting model for the whole subject population was selected.

## Statistical Analysis

After the appropriate model and digit-selection matrix had been selected for each processing type, (addition, subtraction and multiplication), a regression summary record for each subject was generated.

These summary records were then used as input to a program, UNEVEN, for analysis of variance of $\alpha$ and $\beta$ values between groups, sexes, etc. This UNEVEN program was also used for ANOVA runs for error-rates and for MSL values over model values. The program is capable of performing an ANOVA on a data material with an unequal number of observations in the cells.

## Findings

The findings of this paper are discussed as follows: (1) Addition; (2) Subtraction; (3) Multiplication; (4) Educational implications; and (5) Hypotheses for future research.

One of the purposes of this study was to gather data on the processes underlying simple arithmetic and the differences between groups. The reader should be aware that in all cases, Groen and Parkman's (1972) study was used as a precedent and only Ss whose maximal $F$ ($p < .01$), the standard significance of the slope ($\beta$), were used for analysis.

## Addition

Disparate methodology of previous research is partially responsible for causing contradictory findings as to whether the process underlying simple addition is digital or analog. A major criticism of previous research has been the lack of uniform agreement concerning what method, process, or operation was to be used when seeking clarification of the issues involved in the rapid performance of arithmetic facts. Generally speaking, there seemed to be a complete disregard for task analysis in research efforts. Moyer and Landauer (1967), Restle (1970), and Sekuler, Rubin, and Armstrong (1971) used the RT required for judgments of numerical inequality to argue for an analog system. Groen (1967), Suppes and Groen (1967), Suppes, Hyman, and Jerman (1967), Parkman (1971), Parkman and Groen (1971), and Harris and Hamrin (1973) used simple addition in their experiments to argue for a digital system.

The data of the Billington study (1947) and the Knight & Behrens study (1928) also support a digital processing system.

Thomas (1963) assumed that the mental processes underlying simple calculation behave as an information-processing system for which the time needed to perform a given arithmetic operation is strictly proportional to the amount of information handled by the system.

The work of Suppes, Hyman, and Jerman (1967) and Groen (1967) suggest something more complex than a simple memory search, but are ambiguous as to whether the process is digital, or analog. They reported the latencies of adding very small numbers by first graders. They tested several mathematical models and concluded that, in general, the latency depends on the magnitude of the smaller number. This might mean that S selects the larger of the two numbers and then uses the smaller number to control a counting process. For example, Suppes and Groen (1967) in an investigation of the counting algorithms used by first graders found that the latencies were linearly related to the minimum of the two addends. The various models considered by them:

1. The counter is initially set to 0, p is added and then q (Model 1: $x = p + q$).
2. The counter is set to p, the left-most digit, and then q is added (Model 2: $x = q$).
3. The counter is set to q, the right-most digit, and then p is added (Model 3: $x = p$).
4. The counter is set to the minimum of p and q, the maximum is then added (Model 4: $x = MAX(p,q)$.
5. The counter is set to the maximum of p and q, the minimum is then added (Model 5: $x = MIN(p,q)$.
6. Thomas model (Model 6: $x = \log(p + q + r)$.

gave rise to five versions of the equation:

$$RT = \alpha + \beta x$$

where x is a structural variable defined by the model under consideration. The only model to yield a satisfactory fit was the model with minimum digit (MINDIG) as the structural variable (Model 5: $x = MIN(p,q)$. Figure 1 shows RT broken down into processes according to Model 5.

This MINDIG model can be interpreted as follows. The total problem-processing time can be divided into a problem-independent constant time ( $\alpha$) and a problem-dependent variable time ( $\beta$). The $\alpha$-part is made up of the time required to encode the projected problem, select the larger of the two digits and store it in an accumulator, store the smaller digit (MINDIG) in a register and set the counter to zero.

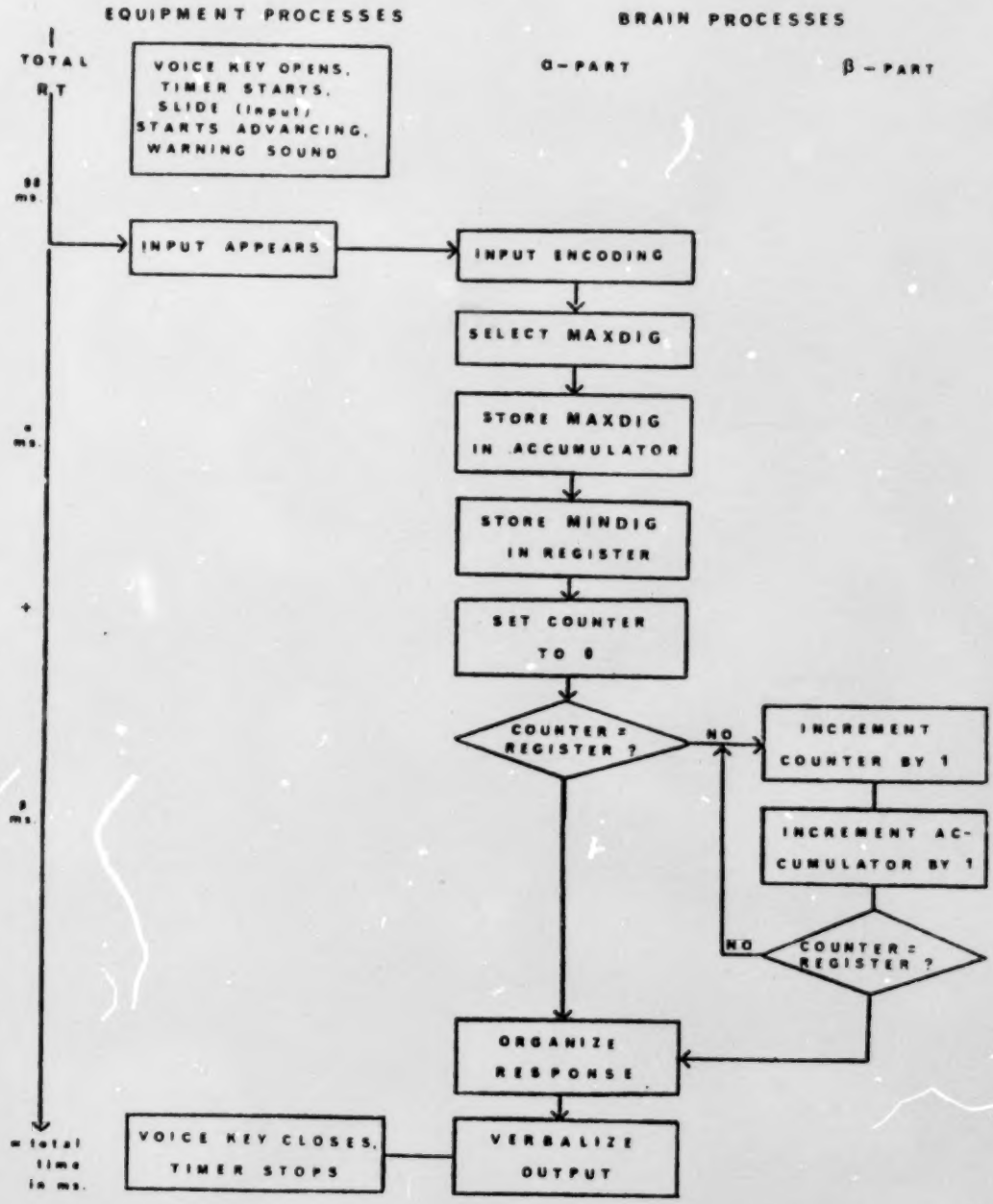


Fig. 1. RT latency broken down into processing stages for addition problems according to Model 5: x = MIN(p,q).

The α-part also includes the time needed to organize and verbalize the response. The β-part consists of the time required to execute the actual three-step counting cycle: (1) comparing the counter to the register, (2) adding one to the accumulator, and (3) adding one to the counter. This cycle is repeated until the counter equals the register. Thus, the time is linearly dependent upon the value of MINDIG.

Considering this background, the most important finding of this study would seem to be that normal fourth graders and MRs display a high positive correlation with Suppes' MINDIG model which strongly supports a digital system (see Figure 2).

The CAs also show a positive correlation with Model 5. It has been hypothesized that simple addition is performed by a memory retrieval operation in adults. One of the purposes of the CA-match was to examine what changes occur as a result of experience and over-learning. The analysis of the data suggests that there is no change in the processes used, except for decrease in both the intercept and the slope. Thus, the shorter RTs of the CA-match may be attributed to faster perceptual and motor processes as well as a faster memory search.

Groen and Parkman (1972) hypothesized that Ss cease to use Model 5 but have memorized the answers which are stored in a rapid-access memory. The slope for the CAs was found to be 41 ms.(Figure 2). Parkman and Groen have further hypothesized that adults degrade to the more childish method of silent counting five per cent of the time which would account for the observed CA slopes. Presumably, the strategy switch would occur if retrieval from fast access memory were to fail. Brown and McNeill's (1966) "tip of the tongue" phenomenon states that such failures to retrieve are common. Thus, it is reasonable to assume that a person confronted with such a failure to retrieve would fall back on a reconstructive algorithm.

In comparing model parameters between different reported experiments, it must be remembered that a large component of the intercept parameter, the α-value, is dependent on the physical set-up of each experiment, including the type of equipment and procedures used to present the problem to S and the way in which S responds (write answer, push-button, voice-key), as well as various constant time delays in the timing equipment.

Therefore, no valid comparisons of α can be made between different experiments. Only within the same experiment, can any valid conclusions be drawn from differences in β between groups.

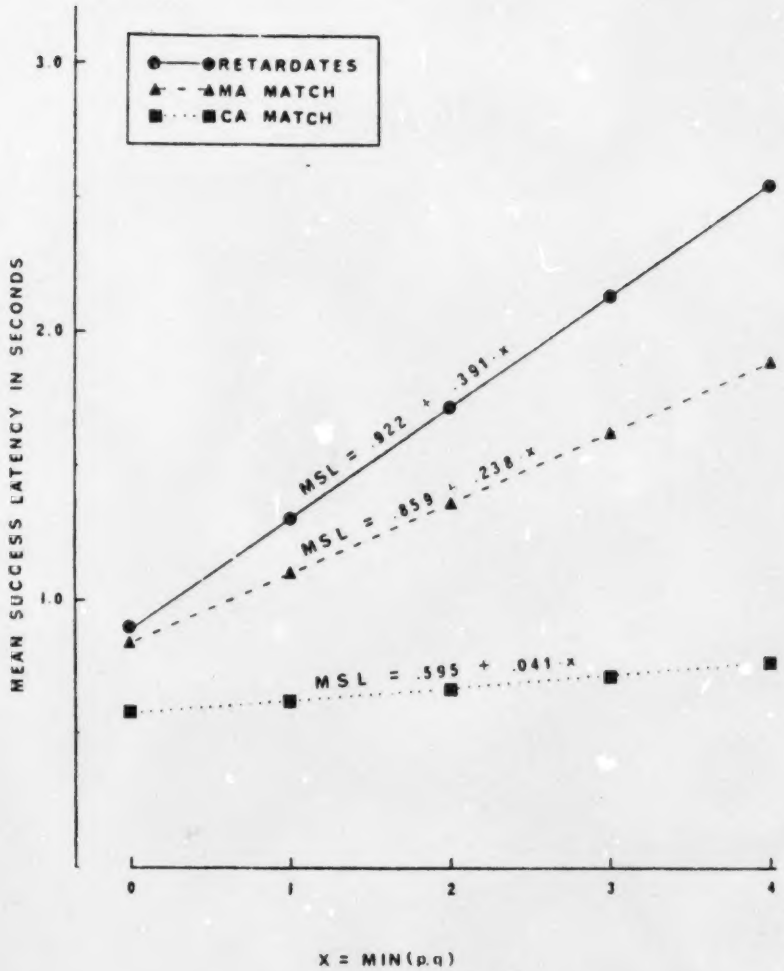


Fig. 2. Regression lines for 60 addition facts analyzed according to Model 5 (N=43).

On the other hand, in a correctly designed experiment, the slope parameter, $\beta$, is entirely equipment-independent and reflects true differences in the mental processing of the different problems. Therefore, the discussion of the different reported addition experiments will be confined to the slope parameter (See Figure 3 & Table 1).

The MAs and the CAs in the present study closely agree with an earlier unpublished study by Harris and Hamrin (See Figure 4). The slopes of the two high school samples were the same order of magnitude as Parkman and Groen's (1971) college students (See Figure 3). The MA group from both studies also agree with Billington's (1947) third graders (See Figure 5).

Groen and Parkman (1972) report $\beta$ =340 ms. for first graders at the end of the school year (N=37, CA=6.10, IQ=125) which differs greatly from Suppes and Groen's (1967) first graders (N=30, no other data given). Suppes and Groen report $\beta$ =710 ms. for first graders tested early in the school year (See Figure 3).

In this study, differences between the RTs of the MA and the MR sample were found to exist completely in the slope. Thus, the longer RTs of the MRs may not be attributed to slower perceptual and motor processes, but to slower memory search or counting procedures. Differences between the RTs of the MA and the CA sample and the MR and the CA sample were found to exist in both the intercept and the slope.

One of the purposes of this study is to investigate how MRs process addition facts in comparison with other groups. The MRs in this study received a $\beta$ =414 ms. This value was much lower than the $\beta$ for Suppes and Groen's first graders but not as low as Groen and Parkman's "very bright" first graders. The MRs in the earlier study (Harris and Hamrin, 1973) received a $\beta$ =920 ms. which is more than twice the $\beta$ for the MR sample used in this study.

Most psychological research in mental retardation has been conducted with institutionalized Ss. While the confounding of institutionalization and retardation has been deplored, its possible consequences have seldom been demonstrated. Among the factors which might conceivably be involved in the inferior performance of institutionalized MRs relative to non-institutionalized MRs of equivalent CA, MA, and IQ are:

1. Variance in MA does not account for all variance in cognitive behavior. Institutionalized MRs may be partially selected on the basis of additional handicaps to those ascribable to low MA. Although data on this point are lacking, MRs may have been selected for institutionalization partly because they were unable to "profit from"

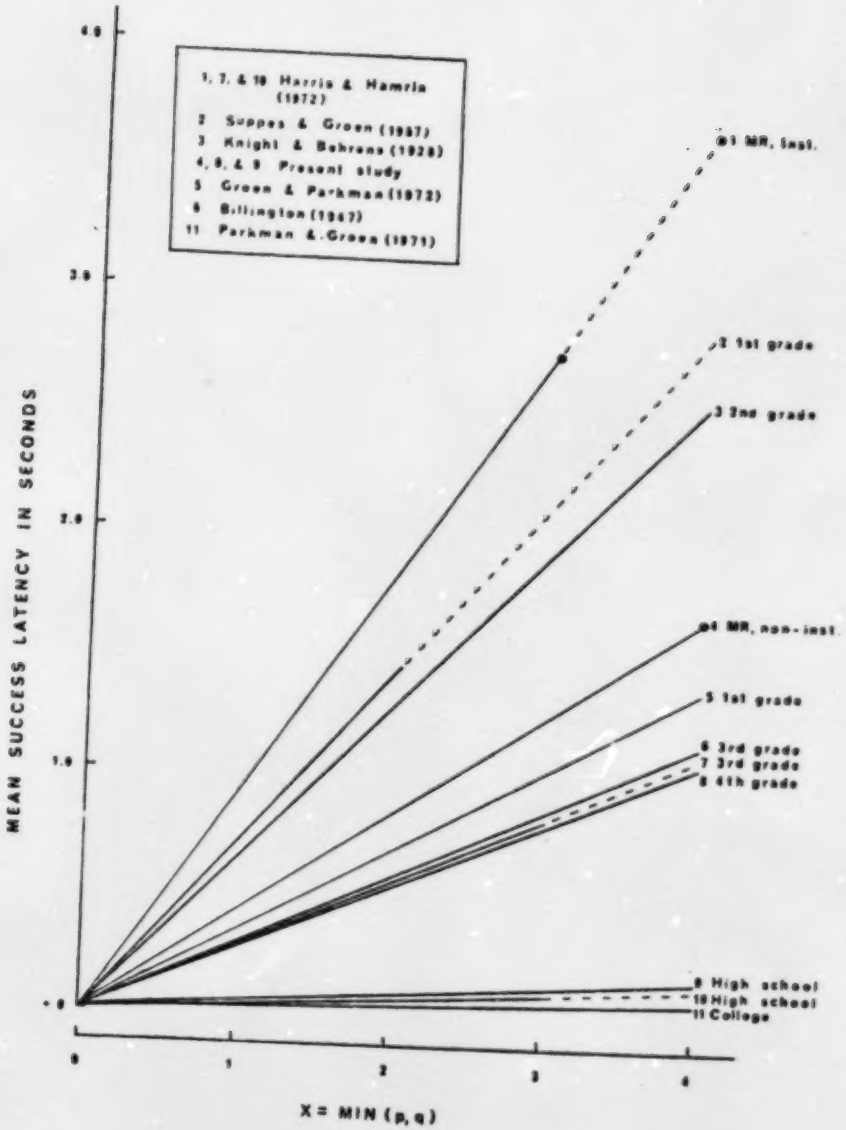
1, 7, & 10 Harris & Hamrin (1972)
2 Suppes & Groen (1967)
3 Knight & Behrens (1928)
4, 8, & 9 Present study
5 Groen & Parkman (1972)
6 Billington (1967)
11 Parkman & Groen (1971)
MEAN SUCCESS LATENCY IN SECONDS
X = MIN (p, q)
1 MR, inst.
2 1st grade
3 2nd grade
4 MR, non-inst.
5 1st grade
6 3rd grade
7 3rd grade
8 4th grade
9 High school
10 High school
11 College

Fig. 3. Slope parameters for seven addition experiments.

Table 1

$\alpha$, $\beta$, r, and F values of MSL

for Seven Addition Studies (Model 5)

| Study | $\alpha$ | $\beta$ | r | F |
|---|---|---|---|---|
| Present Study (N=43) | | | | |
| MR (n=13) | .898 | .414 | .943 | 467.15 |
| MA (n=17) | .841 | .264 | .894 | 230.16 |
| CA (n=13) | .592 | .042 | .740 | 70.25 |
| Harris & Hamrin (N=54) | | | | |
| MR (n=18) | 1.611 | .920 | | |
| MA (n=18) | 1.100 | .269 | | |
| CA (n=18) | .587 | .035 | | |
| Groen & Parkman (N=37) | | | | |
| 1st Grade | 2.53 | .340 | | |
| Suppes & Groen (N=unknown) | | | | |
| 1st Grade | 3.26 | .710 | | |
| Parkman & Groen (N=6) | | | | |
| College | .695 | .020 | | |
| Billington (N=103) | | | | |
| 3rd Grade | 1.432 | .290 | .951 | 28.50 |
| Knight & Behrens (N=25) | | | | |
| 2nd Grade | 1.857 | .644 | .982 | 78.00 |

$F_{.9995}(1,58) = 13.68$

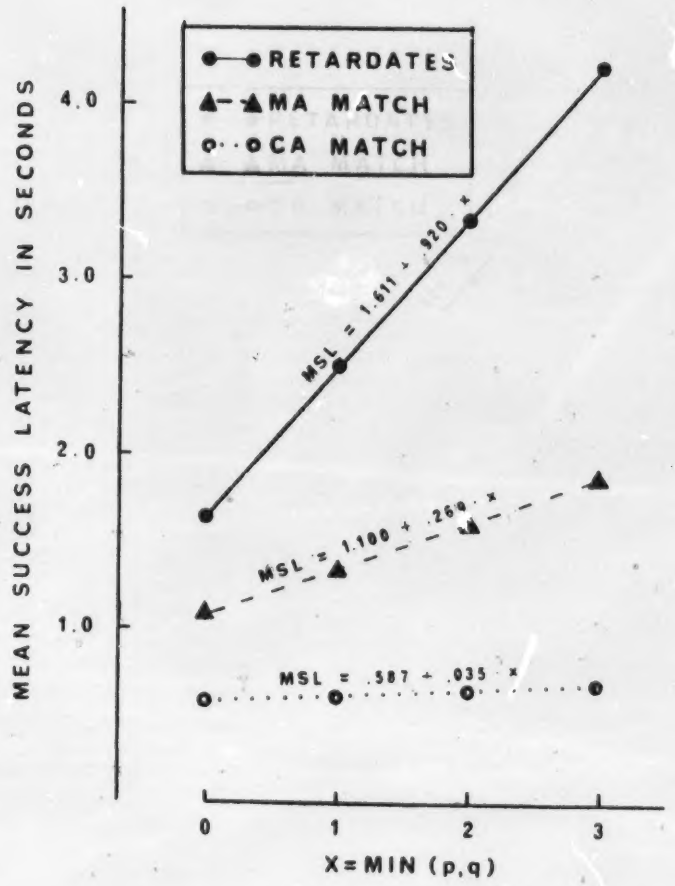


Fig. 4. Harris and Hamrin's data (1973) for addition analyzed according to Model 5 (N=54).

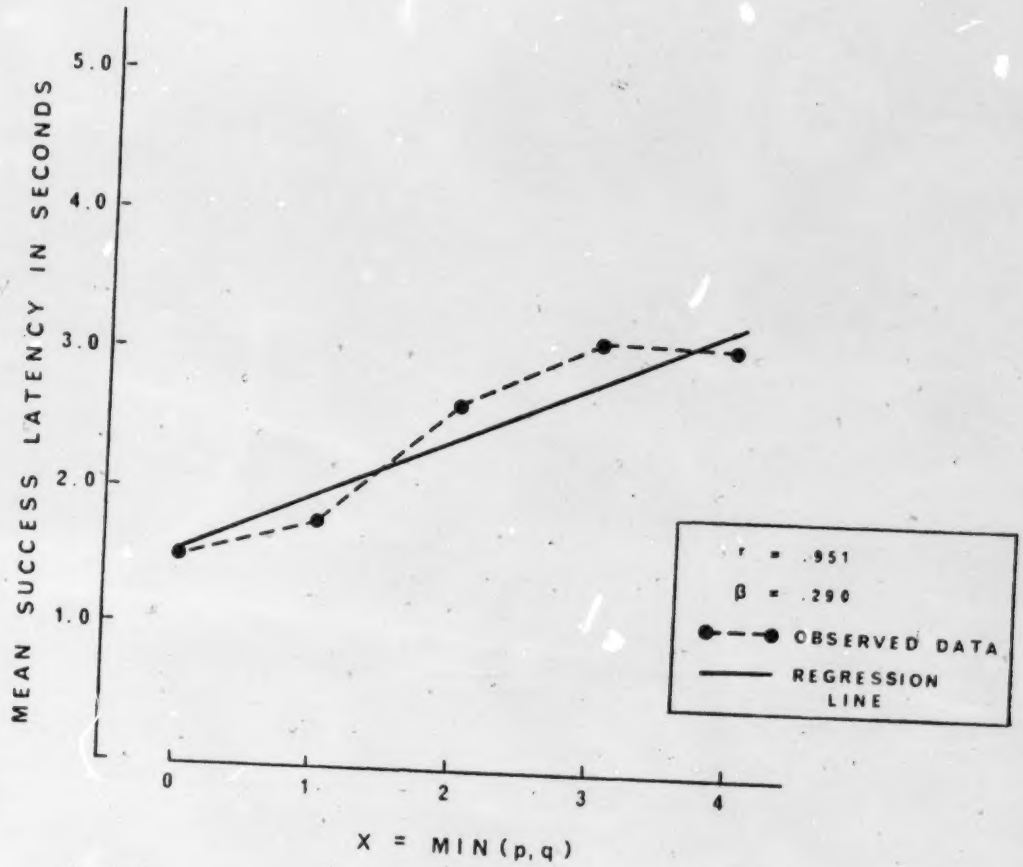


Fig. 5. Billington's data (1947) for addition analyzed according to Model 5 (N=103).

or "adjust to" special classes; and

2. The deprivation and failure involved in institutionalization may foster additional deficient cognitive behavior.

All these studies indicate that they do indeed replicate some kind of a common process and that the slope parameter is a function of mental development and training. The longer RTs of children and MRs as compared with CAs may be attributed to both slower perceptual and motor processes and to slower memory search. These studies all indicate that as CA increases, RTs decrease.

## Subtraction

The only two previously published studies on the subtraction process, Knight and Behrens (1928) and Billington (1947), do not propose that this process is digital nor do they propose the use of any particular model. However, if the addition process is digital, as has been shown in this study as well as earlier studies (Groen, 1967; Suppes and Groen, 1967; Suppes, Hyman, and Jerman, 1967; and Parkman and Groen, 1971), it can then be argued that the subtraction process is also digital. A comparison of the experiment-independent model parameter, $\beta$, shows a good agreement between this study and previous studies (See Figure 6).

The subtraction study was identical to the addition study except that 66 subtraction facts ($p - q \leq 10$, $p \leq 10$,) and ($q \leq 10$) were used as stimuli. The following six models used were: Model 1: $x = p - q$, Model 2: $x = p$, Model 3: $x - q$, Model 4: $x = p + q$, Model 5: $x = \text{MIN} \begin{Bmatrix} p - q \\ q \end{Bmatrix}$, and Model 6: $x = \log(p + q + r)$. Using these six models, the processing and analysis of data was identical to the addition study.

The result of the analysis of the subtraction data shows that the majority of the MR and MA Ss follow the $\text{MIN} \begin{Bmatrix} p - q \\ q \end{Bmatrix}$ model which supports the use of a digital rather than an analog system. An analysis of the Knight and Behrens and the Billington subtraction data (See Figures 7 and 8), following the methodology developed for the present study, shows that the subjects used in those studies also follow the $\text{MIN} \begin{Bmatrix} p - q \\ q \end{Bmatrix}$ model. The model parameters from those studies and from the present study are summarized in Table 2 and Figure 9.

Out of 18 Ss in the CA group only two followed Model 5, four followed Model 1, and one followed Model 2. The remaining 11 Ss did not follow any of the six proposed models. However, all the CA Ss have a MSL over all the subtraction problems that is far below the MSL for the MR and MA groups and only 22 per cent above the MSL for the addition process (See Table 3). This supports the fact that the CA Ss utilize a memory-retrieval system for processing subtraction

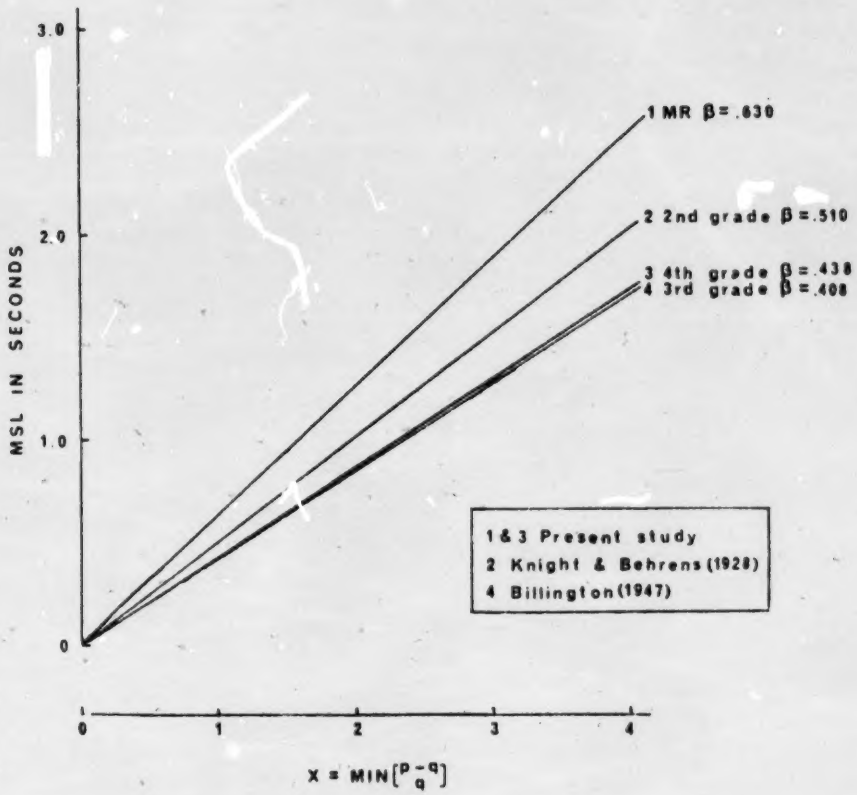


Fig. 6. Slope parameters for three subtraction studies.

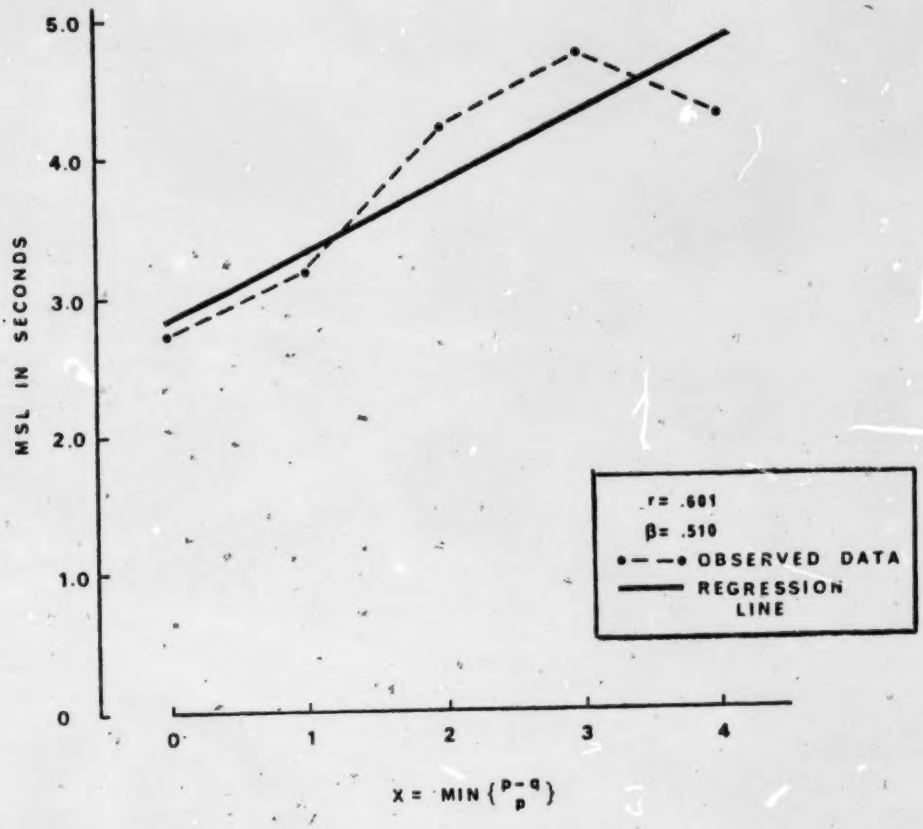


Fig. 7. Knight and Behrens' data (1928) for subtraction analyzed according to Model 5 (N=15).

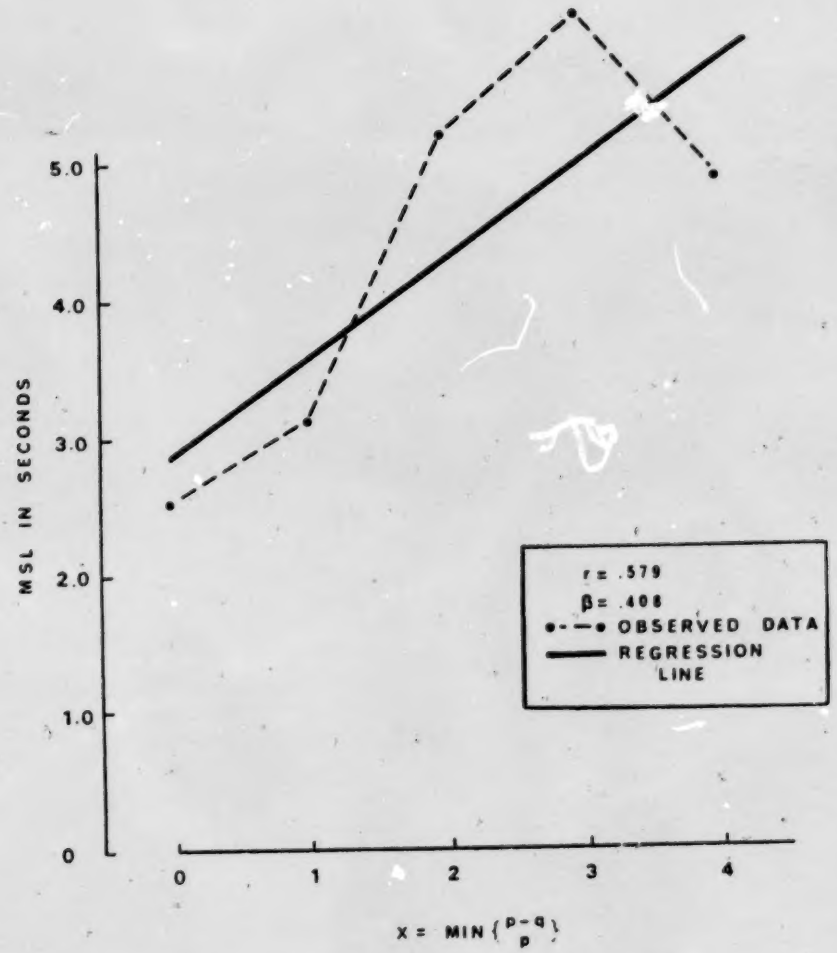


Fig. 8. Billington's data (1947) for subtraction analyzed according to Model 5 (N=103).

Table 2

$\alpha$, $\beta$, r, and $\underline{F}$ Values of MSL for Three Subtraction Studies (Model 5)

| Study | $\alpha$ | $\beta$ | r | $\underline{F}$ |
|---|---|---|---|---|
| Present Study (N=26) | | | | |
| MR (n=14) | 1.473 | 0.630 | .827 | 134.21 |
| MA (n=12) | 1.446 | 0.438 | .771 | 90.89 |
| Knight and Behrens (N=15) | | | | |
| 2nd Grade | 2.845 | 0.510 | .601 | 34.50 |
| Billington (N=103) | | | | |
| 3rd Grade | 1.750 | 0.408 | .579 | 26.69 |

$F_{.9995}(1,62) = 13.57$

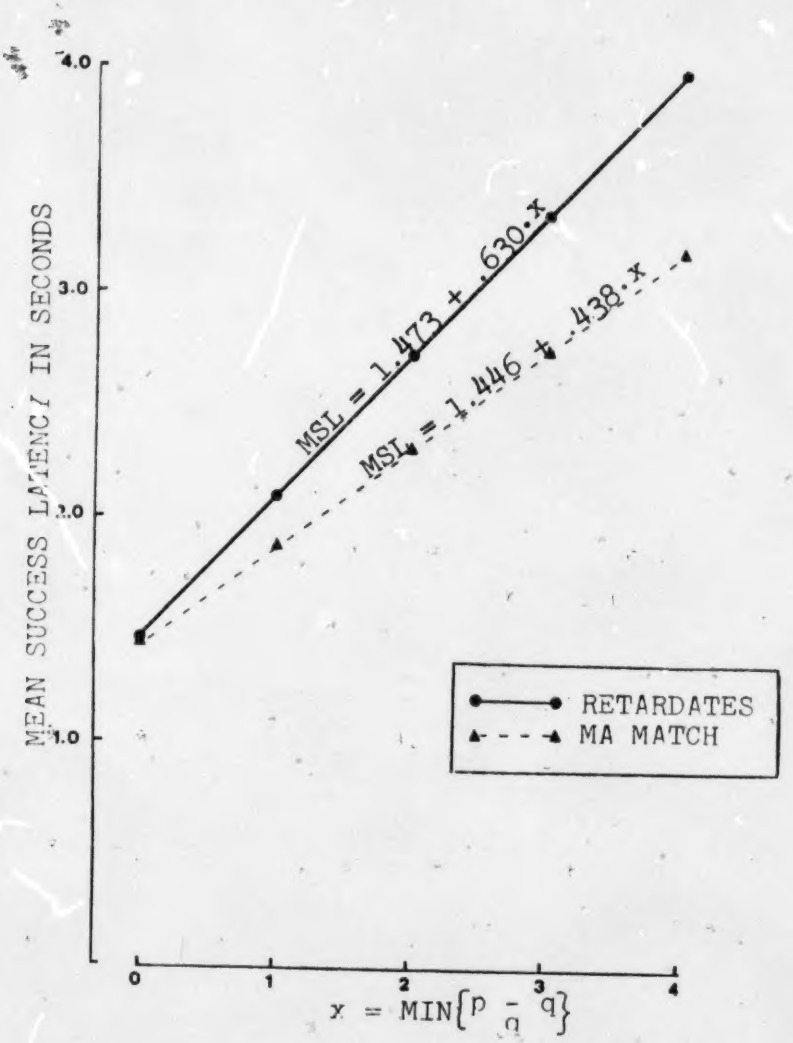


Fig. 9. Regression lines for 64 subtraction facts analyzed according to Model 5 (N=26).

problems, analogously to their addition processing. This is true perhaps to a larger extent for subtraction problems than for addition problems as the subtraction process seems to be more difficult.

Table 3

Average MSL for All Problems

| Group | Addition | Subtraction | Multiplication |
|---|---|---|---|
| MR (selected) | 1.690 | 2.660 | 2.212 |
| MA (selected) | 1.310 | 2.212 | 1.583 |
| CA (selected) | 0.672 | 0.829* | 0.670 |

*18 Ss were used

Multiplication

Parkman (1972) found in his study of the multiplication processes of six college students that adults show a high degree of similarity between their processing of addition and multiplication problems. Parkman's subjects showed a strong adherence to the MINDIG (x = MIN (p, q) model. However, he suggests that the low value of the slope parameter, $\beta$ =019 ms., is due to a memory-retrieval process rather than a computational algorithmic process.

The multiplication study was designed to yield performance models used in solving 36 simple multiplication ($p \times q \leq 25$, $p \leq 0$, and $q \leq 0$) problems.

The following solution models were used:

1. The latency depends on the size of the product (Model 1: $x = p \times q$).
2. The latency depends on the size of the left-most digit (Model 2: $x = p$).
3. The latency depends on the size of the right-most digit (Model 3: $x = q$).
4. The latency depends on the larger of p and q (Model 4: $x = MAX(p,q)$).
5. The latency depends on the smaller of p and q (Model 5: $x = MIN(p,q)$).
6. The latency depends on the amount of information handled by the system (Model 6: $x = \log(p + q + r)$).

Using the above six models, data analysis was carried out identical with the addition study.

The results of this investigation concur with Parkman's results in that the CAs show a strong correlation between MSL and the MINDIG model and a high degree of similarity between their multiplication and addition processes. (See Figure 10). The slope parameter, $\beta$ =045 ms. for multiplication is very close to the slope parameter, $\beta$ =042 ms., for addition. As discussed earlier this $\beta$ -value is too low to account for a strict computational process. A possible explanation is that adults use a memory retrieval system and only occasionally default to a counting process.

For simple multiplication, Parkman (1972) found that RTs for ties did not systematically differ from non-tie problems, at least not in the manner found by Parkman and Groen (1971) for addition. The results of the present study support this finding. Hence, in multiplication, it may be concluded that ties and non-ties are performed by identical processes, whereas in addition such problems may be performed quite differently from one another.

The MR and the MA groups also show a high adherence to the MINDIG model. The MRs' average slope parameter is very close to their addition parameter. The MAs' average slope parameter is approximately 20 per cent larger than for their addition process. One possible explanation is that the MAs have just started learning multiplication whereas the MRs have had a longer training period.

In conclusion, the results of this study support the theory that addition and multiplication are based on the same common mental processing mechanism.

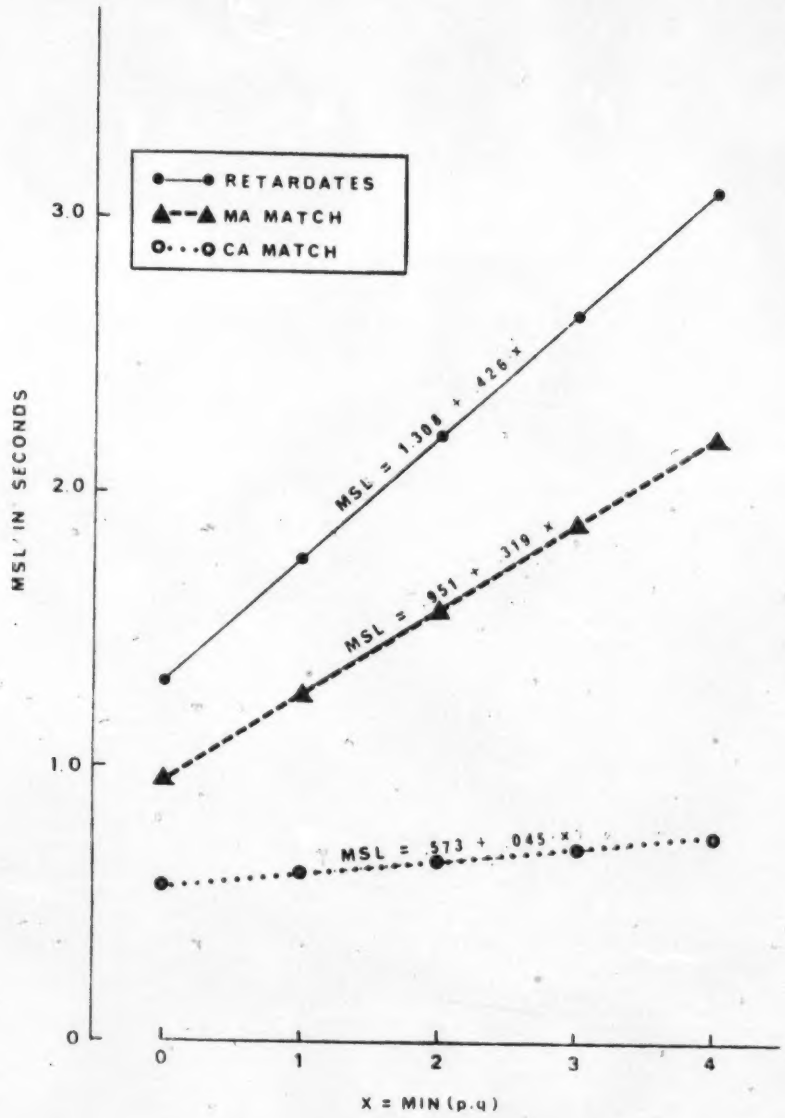


Fig. 10 Regression lines for 34 multiplication facts analyzed according to Model 5 (N=35).

## Educational Implications

The results of this study establish that the majority of Ss process number facts by utilizing a particular model. From a psychological standpoint, the most suggestive single finding is the importance of memory as a reconstructive process in solving simple arithmetic problems, at least for the MAs and MRs.

Heretofore, relatively little attention has been directed toward investigating factors of arithmetic performance among the mentally retarded. Cruickshank (1948) has reported that, compared with the intellectually normal of the same mental age, the retarded are reasonably alike in areas of computation, although they are more careless than the normal and exhibit a greater incidence of "primitive behavior" such as counting on their fingers. The error analysis done in this paper would refute the notion that MRs are more careless. The notion of finger counting is strongly supported by direct observation and is reflected in the longer RTs of the MRs. In the addition and subtraction experiments, the highest number handled was ten which allowed for ease in finger counting. This suggests that the retarded do not handle the abstract symbols mechanically but prefer to manipulate objects.

Some of the implications for teaching the retarded that emerge include the following:

1. The manipulation of objects and active participation by the children to form the foundation for understanding numbers.

2. The manipulation of objects in game-like situations involving as many sense modalities as possible.

3. The development of a method which would allow for the counting of fixed reference points on numbers. The patterns should approximate the common patterns found on dice, dominoes, playing cards, and beginning arithmetic texts.

Theoretically, the pattern would do two things: help to stabilize perception and allow the MR to work with the whole number because the numeral would be combined with its value.

## Suggestions for Future Research

Some of the results considered peripheral to this study may provide grist for future research suggestions. These include:

1. Explore the variability within the MR group to contribute to our understanding of cognitive development in MRs.

2. Increase the number of subjects involved in order to provide more data to make more meaningful inferences about individual differences or developmental processes.

3. Observe the changes in $\alpha$ , $\beta$ , and errors in the same sample over a period of years in order to estimate parameters that would reflect the course of developmental change in the performance of arithmetic tasks. Does the teaching of an algorithm help illuminate concepts? What is the optimum time to learn a specific topic or skill?

4. Analyze the MSL data by applying more complex, non-linear models that could possibly explain the slight RT drop-off for the highest value of x. What variables predict performance in simple arithmetic tasks? What per cent of the variance in total performance is accounted for by each of the variables?

5. Compare each process within each subject which would give insight into the memory load required by different processes. Would mnemonic devices be of use where memory load is heavy?

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## Appendix A

### List of terms

α: a constant which represents the combined durations of all brain processes such as perception, encoding, etc., intercept.

analog: an associational measuring or a continuous record, performs mathematical problems by transforming data into measures.

ANOVA: analysis of variance.

β: a variable stage which can be decomposed into a number of identical steps, slope.

CA: chronological age.

chronometry: the art of measuring time accurately, the measuring of time by periods or divisions.

digital: counting, operates directly with the data without regard for magnitude.

EPD: electric potential differential.

F: the standard statistic for the significance of the slope parameter β, ratio between two variances.

latency: the time from the onset of the stimulus to the initiation of a response.

MA: mental age.

MAXDIG: the larger of two digits.

memory, reconstructive process of: the generation of facts on the basis of stored rules.

memory, reproductive process of: the retrieval of stored facts.

MINDIG: the smaller of two digits.

monotonically: unvarying, lacking in variety.

MR: mentally retarded

ms.: millisecond.

MSL: mean success latency, this is the average RT over all successful answers for a particular problem. Missing data and outliers are excluded from MSL. It does not include the equipment dependent time delay of 980 msec.

MSL MATRIX: table of the MSL for all selected problems for each subject and process.

mV: millivolt.

ORDER VECTOR: A vector defining the order in which the slides were shown to the subject. For instance, an order vector: (3,1,6,4,2,5) indicates that the second column of the RT MATRIX contains the slide series shown first, the fifth column the second series, the first column the third series shown, etc.

outlier: having an excessively long response latency.

PROBLEM MATRIX: A table of the displayed problems. A particular element in the PROBLEM MATRIX contains the problem matching the RT in the corresponding element in the RT MATRIX.

r: correlation coefficient.

regression analysis: considers the frequency distributions of one variable when another is held fixed at each of several levels.

RT: reaction time as measured by the electronic timer and recorded by the experimenter. RTs do include an equipment-dependent time constant of 980 msec.

RT MATRIX: Table of RT in the format they were recorded by the experimenter; i.e. there is one RT MATRIX for each subject and process.

Split: the magnitude of difference between two digits.

stochastic: based on one item in the probability distribution of an ordered set of observations, conjectural.

ties: addition problems with equal addends, subtraction problems with remainder equal to 0, and multiplication problems where multiplier and multiplicand are equal.

# Appendix B

LISTING OF PROBLEMS:

| TRAY NO. | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1: | 7+ 0.... | 3+ 3.... | 3+ 0.... | 4+ 6.... | 0+ 0.... | 0+ 0.... |
| 2: | 9+ 0.... | 7+ 1.... | 1+ 6.... | 0+ 3.... | 0+ 0.... | 0+ 0.... |
| 3: | 6+ 2.... | 4+ 6.... | 0+ 0.... | 3+ 3.... | 0+ 0.... | 0+ 0.... |
| 4: | 0+ 4.... | 1+ 1....10+ | 0.... | 7+ 0.... | 0+ 0.... | 0+ 0.... |
| 5: | 8+ 2.... | 0+ 3.... | 5+ 4.... | 0+ 4.... | 0+ 0.... | 0+ 0.... |
| 6: | 1+ 4.... | 9+ 0.... | 4+ 1.... | 8+ 2.... | 0+ 0.... | 0+ 0.... |
| 7: | 0+ 0.... | 1+ 8.... | 7+ 3.... | 4+ 0.... | 0+ 0.... | 0+ 0.... |
| 8: | 1+ 3.... | 0+ 9.... | 0+ 1.... | 3+ 7.... | 0+ 0.... | 0+ 0.... |
| 9: | 4+ 2.... | 0+ 7.... | 1+ 0.... | 0+ 2.... | 0+ 0.... | 0+ 0.... |
| 10: | 1+ 5.... | 2+ 3.... | 5+ 1.... | 6+ 1.... | 0+ 0.... | 0+ 0.... |
| 11: | 0+ 5.... | 3+ 8.... | 0+10.... | 0+ 6.... | 0+ 0.... | 0+ 0.... |
| 12: | 0+ 9.... | 4+ 5.... | 4+ 5.... | 1+ 5.... | 0+ 0.... | 0+ 0.... |
| 13: | 5+ 1.... | 0+ 4.... | 7+ 1.... | 8+ 0.... | 0+ 0.... | 0+ 0.... |
| 14: | 2+ 6.... | 6+ 0.... | 6+ 0.... | 2+ 5.... | 0+ 0.... | 0+ 0.... |
| 15: | 3+ 4.... | 7+ 0.... | 2+ 2.... | 0+ 9.... | 0+ 0.... | 0+ 0.... |
| 16: | 0+10.... | 2+ 8.... | 2+ 0.... | 1+ 4.... | 0+ 0.... | 0+ 0.... |
| 17: | 3+ 7.... | 3+ 7.... | 0+ 5.... | 5+ 5.... | 0+ 0.... | 0+ 0.... |
| 18: | 8+ 0.... | 4+ 0.... | 2+ 3.... | 9+ 1.... | 0+ 0.... | 0+ 0.... |
| 19: | 4+ 5.... | 3+ 5.... | 6+ 2.... | 3+ 6.... | 0+ 0.... | 0+ 0.... |
| 20: | 1+ 7.... | 8+ 0.... | 3+ 2.... | 5+ 2.... | 0+ 0.... | 0+ 0.... |
| 21: | 2+ 1.... | 6+ 4.... | 1+ 1.... | 2+ 6.... | 0+ 0.... | 0+ 0.... |
| 22: | 9+ 1.... | 1+ 5.... | 1+ 7.... | 3+ 1.... | 0+ 0.... | 0+ 0.... |
| 23: | 7+ 3.... | 1+ 2.... | 4+ 3.... | 2+ 7.... | 0+ 0.... | 0+ 0.... |
| 24: | 5+ 4.... | 4+ 1.... | 4+ 2.... | 5+ 0.... | 0+ 0.... | 0+ 0.... |
| 25: | 8+ 1.... | 5+ 4.... | 1+ 9.... | 8+ 1.... | 0+ 0.... | 0+ 0.... |
| 26: | 10+ 0.... | 2+ 6.... | 9+ 0.... | 1+ 3.... | 0+ 0.... | 0+ 0.... |
| 27: | 2+ 0.... | 6+ 1.... | 4+ 4.... | 2+ 1.... | 0+ 0.... | 0+ 0.... |
| 28: | 4+ 4.... | 1+ 0.... | 0+ 7.... | 6+ 3.... | 0+ 0.... | 0+ 0.... |
| 29: | 0+ 3.... | 0+ 6.... | 1+ 8.... | 5+ 3.... | 0+ 0.... | 0+ 0.... |
| 30: | 2+ 8.... | 4+ 2.... | 6+ 4.... | 3+ 0.... | 0+ 0.... | 0+ 0.... |
| 31: | 7+ 1.... | 2+ 0.... | 4+ 4.... | 7+ 2.... | 0+ 0.... | 0+ 0.... |
| 32: | 1+ 9.... | 1+ 6.... | 2+ 8.... | 1+ 6.... | 0+ 0.... | 0+ 0.... |
| 33: | 6+ 1.... | 4+ 3.... | 2+ 4.... | 3+ 5.... | 0+ 0.... | 0+ 0.... |
| 34: | 3+ 2.... | 2+ 4.... | 1+ 2.... | 0+ 0.... | 0+ 0.... | 0+ 0.... |
| 35: | 4+ 1.... | 4+ 4.... | 0+ 8.... | 1+ 2.... | 0+ 0.... | 0+ 0.... |
| 36: | 3+ 6.... | 6+ 3.... | 3+ 5....10+ | 0.... | 0+ 0.... | 0+ 0.... |
| 37: | 0+ 2....10+ | 0.... | 7+ 2.... | 0+ 8.... | 0+ 0.... | 0+ 0.... |
| 38: | 5+ 0.... | 5+ 2.... | 5+ 3.... | 5+ 4.... | 0+ 0.... | 0+ 0.... |
| 39: | 1+ 2.... | 7+ 3.... | 6+ 3.... | 2+ 4.... | 0+ 0.... | 0+ 0.... |
| 40: | 3+ 3.... | 0+ 2.... | 2+ 1.... | 4+ 1.... | 0+ 0.... | 0+ 0.... |
| 41: | 6+ 4.... | 3+ 2.... | 1+ 3.... | 7+ 3.... | 0+ 0.... | 0+ 0.... |
| 42: | 4+ 0.... | 1+ 4.... | 8+ 1.... | 2+ 8.... | 0+ 0.... | 0+ 0.... |
| 43: | 6+ 0.... | 5+ 3.... | 5+ 0.... | 0+ 1.... | 0+ 0.... | 0+ 0.... |
| 44: | 3+ 1.... | 9+ 1.... | 2+ 7.... | 3+ 4.... | 0+ 0.... | 0+ 0.... |
| 45: | 1+ 8.... | 3+ 4.... | 3+ 1.... | 1+ 0.... | 0+ 0.... | 0+ 0.... |
| 46: | 1+ 0.... | 5+ 5.... | 2+ 6.... | 6+ 4.... | 0+ 0.... | 0+ 0.... |
| 47: | 2+ 4.... | 1+ 9.... | 5+ 2.... | 5+ 1.... | 0+ 0.... | 0+ 0.... |
| 48: | 1+ 6.... | 2+ 5.... | 3+ 6.... | 1+ 8.... | 0+ 0.... | 0+ 0.... |
| 49: | 4+ 3.... | 5+ 1.... | 9+ 1.... | 0+10.... | 0+ 0.... | 0+ 0.... |
| 50: | 1+ 1.... | 5+ 0.... | 5+ 5.... | 0+ 7.... | 0+ 0.... | 0+ 0.... |
| 51: | 7+ 2.... | 2+ 2.... | 1+ 4.... | 4+ 5.... | 0+ 0.... | 0+ 0.... |
| 52: | 2+ 2.... | 2+ 1.... | 2+ 5.... | 4+ 4.... | 0+ 0.... | 0+ 0.... |
| 53: | 4+ 6.... | 0+ 0.... | 1+ 5.... | 9+ 0.... | 0+ 0.... | 0+ 0.... |
| 54: | 3+ 0.... | 2+ 7.... | 0+ 9.... | 7+ 1.... | 0+ 0.... | 0+ 0.... |
| 55: | 5+ 2.... | 8+ 2.... | 6+ 1.... | 1+ 9.... | 0+ 0.... | 0+ 0.... |
| 56: | 5+ 5.... | 1+ 3.... | 3+ 7.... | 6+ 0.... | 0+ 0.... | 0+ 0.... |
| 57: | 6+ 3.... | 6+ 2.... | 0+ 2.... | 4+ 2.... | 0+ 0.... | 0+ 0.... |
| 58: | 5+ 3.... | 1+ 7.... | 0+ 6.... | 2+ 2.... | 0+ 0.... | 0+ 0.... |
| 59: | 0+ 7.... | 2+ 1.... | 8+ 0.... | 4+ 3.... | 0+ 0.... | 0+ 0.... |
| 60: | 3+ 5.... | 0+ 5.... | 4+ 0.... | 2+ 0.... | 0+ 0.... | 0+ 0.... |
| 61: | 2+ 7.... | 8+ 1.... | 8+ 2.... | 1+ 7.... | 0+ 0.... | 0+ 0.... |
| 62: | 0+ 6.... | 0+ 1.... | 0+ 4.... | 0+ 5.... | 0+ 0.... | 0+ 0.... |
| 63: | 0+ 1.... | 0+10.... | 7+ 0.... | 1+ 1.... | 0+ 0.... | 0+ 0.... |
| 64: | 2+ 5.... | 7+ 2.... | 3+ 3.... | 2+ 3.... | 0+ 0.... | 0+ 0.... |
| 65: | 0+ 8.... | 0+ 8.... | 0+ 3.... | 3+ 2.... | 0+ 0.... | 0+ 0.... |
| 66: | 2+ 3.... | 3+ 0.... | 4+ 6.... | 6+ 2.... | 0+ 0.... | 0+ 0.... |

Exhibit 1.

# Appendix B (con'd)

SUBJECT ID ...A002 LISTING OF INPUT DATA

| TRIAL NO | 1 | | 2 | | 3 | | 4 | | 0 | | 0 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LINE NO. | PR | TIME | PR | TIME | PR | TIME | PR | TIME | PR | TIME | PR | TIME |
| 1 | 7+ 0 | 1.76 | 3+ 3 | 1.73 | 3+ 0 | 1.65 | 4+ 6 | 1.79 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 2 | 9+ 0 | 1.68 | 7+ 1 | 1.70 | 1+ 6 | 2.06 | 0+ 3 | 1.87 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 3 | 6+ 2 | 2.10 | 4+ 6 | 1.92 | 0+ 0 | 1.92 | 3+ 3 | 1.73 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 4 | 0+ 4 | 1.61 | 1+ 1 | 1.79 | 10+ 0 | 1.81 | 7+ 0 | 1.80 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 5 | 8+ 2 | 1.95 | 0+ 3 | 1.79 | 5+ 5 | 1.85 | 0+ 4 | 1.72 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 6 | 1+ 4 | 1.85 | 9+ 0 | 1.71 | 4+ 1 | 1.79 | 8+ 2 | 1.74 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 7 | 0+ 0 | 2.17 | 1+ 8 | 1.79 | 7+ 3 | 1.91 | 4+ 0 | 1.69 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 8 | 1+ 3 | 1.86 | 0+ 9 | 1.73 | 0+ 1 | 1.71 | 3+ 7 | 1.79 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 9 | 4+ 2 | 1.75 | 0+ 7 | 1.56 | 1+ 0 | 1.73 | 0+ 2 | 1.65 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 10 | 1+ 5 | 2.12 | 2+ 3 | ***** | 5+ 1 | 1.78 | 6+ 1 | 1.66 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 11 | 0+ 5 | 1.67 | 3+ 6 | 1.97 | 0+10 | 1.83 | 0+ 6 | 1.74 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 12 | 0+ 9 | 1.73 | 4+ 5 | 1.89 | 4+ 5 | 1.78 | 1+ 5 | 1.91 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 13 | 5+ 1 | 1.87 | 0+ 4 | 1.61 | 7+ 1 | 2.01 | 8+ 0 | 1.72 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 14 | 2+ 6 | 1.88 | 6+ 0 | 1.74 | 6+ 0 | 1.84 | 2+ 5 | 1.91 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 15 | 3+ 4 | 1.98 | 7+ 0 | 1.7. | 2+ 2 | 1.84 | 0+ 9 | 1.62 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 16 | 0+10 | 1.99 | 2+ 8 | 1.88 | 2+ 0 | 1.78 | 1+ 4 | 1.73 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 17 | 3+ 7 | 1.93 | 3+ 7 | 1.77 | 0+ 5 | 1.80 | 5+ 5 | 1.75 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 18 | 8+ 0 | 1.48 | 4+ 0 | 1.67 | 2+ 3 | 1.88 | 9+ 1 | 1.87 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 19 | 4+ 5 | 2.22 | 3+ 5 | 2.15 | 6+ 2 | 2.15 | 3+ 6 | 2.30 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 20 | 1+ 7 | 1.36 | 8+ 0 | 1.84 | 3+ 2 | 1.81 | 5+ 2 | 1.83 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 21 | 2+ 1 | 1.98 | 6+ 4 | 1.83 | 1+ 1 | 1.79 | 2+ 6 | ***** | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 22 | 9+ 1 | 1.79 | 1+ 5 | 1.83 | 1+ 7 | 2.03 | 3+ 1 | 1.87 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 23 | 7+ 3 | 1.96 | 1+ 2 | 2.05 | 4+ 3 | 1.88 | 2+ 7 | 1.93 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 24 | 5+ 4 | 1.91 | 4+ 1 | 1.92 | 4+ 2 | 2.07 | 5+ 0 | 1.85 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 25 | 8+ 1 | 1.90 | 5+ 4 | 1.99 | 1+ 9 | 1.96 | 8+ 1 | 1.83 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 26 | 10+ 0 | 1.81 | 2+ 6 | 2.21 | 9+ 0 | 1.91 | 1+ 3 | 1.68 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 27 | 2+ 0 | 1.83 | 6+ 1 | 1.79 | 4+ 4 | 1.98 | 2+ 1 | 1.88 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 28 | 4+ 4 | 1.92 | 1+ 0 | 1.84 | 0+ 7 | 1.77 | 6+ 3 | 1.80 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 29 | 0+ 3 | 1.71 | 0+ 6 | 1.79 | 1+ 8 | 1.85 | 5+ 3 | 3.36 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 30 | 2+ 8 | 1.82 | 4+ 2 | 1.96 | 6+ 4 | 1.83 | 3+ 0 | 1.81 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 31 | 7+ 1 | 1.98 | 2+ 0 | 1.85 | 3+ 4 | 1.72 | 7+ 2 | 1.87 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 32 | 1+ 9 | 1.89 | 1+ 6 | 1.97 | 2+ 8 | 1.94 | 1+ 6 | 1.95 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 33 | 6+ 1 | 1.80 | 4+ 3 | 1.73 | 2+ 4 | 1.73 | 3+ 5 | 1.72 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 34 | 3+ 2 | 1.91 | 2+ 4 | 1.98 | 1+ 2 | 1.99 | 0+ 0 | 1.87 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 35 | 4+ 1 | 1.94 | 4+ 4 | 1.98 | 0+ 8 | 1.65 | 1+ 2 | 1.91 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 36 | 3+ 6 | 2.11 | 6+ 3 | ***** | 3+ 5 | 1.93 | 10+ 0 | 1.84 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 37 | 0+ 2 | 1.80 | 10+ 0 | 1.95 | 7+ 2 | 1.80 | 0+ 8 | 1.69 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 38 | 5+ 0 | 1.63 | 5+ 2 | 1.97 | 5+ 3 | 1.83 | 5+ 4 | 1.80 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 39 | 1+ 2 | 1.94 | 7+ 3 | 1.98 | 6+ 3 | 2.02 | 2+ 4 | 2.03 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 40 | 3+ 3 | 1.87 | 0+ 2 | 1.81 | 2+ 1 | 1.78 | 4+ 1 | 1.86 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 41 | 6+ 4 | ***** | 3+ 2 | 1.99 | 1+ 3 | 1.94 | 7+ 3 | 2.02 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 42 | 4+ 0 | 1.58 | 1+ 4 | 1.88 | 8+ 1 | 1.77 | 2+ 8 | 1.81 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 43 | 6+ 0 | 1.65 | 5+ 3 | 1.90 | 5+ 0 | 1.88 | 0+ 1 | 1.68 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 44 | 3+ 1 | 1.74 | 9+ 1 | 2.01 | 2+ 7 | 1.92 | 3+ 4 | 1.78 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 45 | 1+ 8 | 1.79 | 3+ 4 | 2.01 | 3+ 1 | 2.13 | 1+ 0 | 1.64 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 46 | 1+ 0 | 1.69 | 5+ 5 | 1.91 | 2+ 6 | 1.96 | 6+ 4 | 1.89 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 47 | 2+ 4 | 1.98 | 1+ 9 | 1.86 | 5+ 2 | 1.98 | 5+ 1 | ***** | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 48 | 1+ 6 | 1.68 | 2+ 5 | 2.07 | 3+ 6 | 2.00 | 1+ 8 | 2.29 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 49 | 4+ 3 | 1.85 | 5+ 1 | 1.90 | 9+ 1 | ***** | 0+10 | 1.76 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 50 | 1+ 1 | 1.89 | 5+ 0 | 1.70 | 5+ 5 | 1.76 | 0+ 7 | 1.78 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 51 | 7+ 2 | 2.05 | 2+ 2 | 1.81 | 1+ 4 | 1.82 | 4+ 5 | 1.78 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 52 | 2+ 2 | 1.77 | 3+ 1 | 2.01 | 2+ 5 | 1.97 | 4+ 4 | 1.80 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 53 | 4+ 6 | 1.79 | 0+ 0 | 1.96 | 1+ 5 | 2.02 | 9+ 0 | 1.81 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 54 | 3+ 0 | 1.77 | 2+ 7 | ***** | 0+ 9 | 1.83 | 7+ 1 | 1.83 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 55 | 5+ 2 | 2.10 | 8+ 2 | 1.77 | 6+ 1 | 1.81 | 1+ 9 | 1.92 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 56 | 5+ 5 | 1.84 | 1+ 3 | 1.81 | 3+ 7 | 1.78 | 6+ 0 | 1.80 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 57 | 6+ 3 | 2.03 | 6+ 2 | 2.12 | 0+ 2 | 1.86 | 4+ 2 | 2.08 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 58 | 5+ 3 | 2.02 | 1+ 7 | 1.68 | 0+ 6 | 1.98 | 2+ 2 | 1.76 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 59 | 0+ 7 | 1.87 | 2+ 1 | 1.92 | 8+ 0 | 1.71 | 4+ 3 | 1.80 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 60 | 3+ 5 | 1.89 | 0+ 5 | 1.65 | 4+ 0 | 1.72 | 2+ 0 | 1.87 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 61 | 2+ 7 | 1.90 | 8+ 1 | 2.03 | 8+ 2 | 1.78 | 1+ 7 | 1.98 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 62 | 0+ 6 | 1.73 | 0+ 1 | 1.72 | 0+ 4 | 1.64 | 0+ 5 | 1.75 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 63 | 0+ 1 | 1.89 | 0+10 | 1.84 | 7+ 0 | 1.69 | 1+ 1 | 1.95 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 64 | 2+ 5 | 1.79 | 7+ 2 | ***** | 3+ 3 | 1.81 | 2+ 3 | 2.04 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 65 | 0+ 8 | 1.61 | 0+ 8 | 2.12 | 0+ 3 | 1.80 | 3+ 2 | 2.05 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |
| 66 | 2+ 3 | 2.08 | 3+ 0 | 1.79 | 4+ 6 | 1.91 | 6+ 2 | 1.89 | 0+ 0 | 0.00 | 0+ 0 | 0.00 |

Exhibit 2.

## Appendix B (con'd)

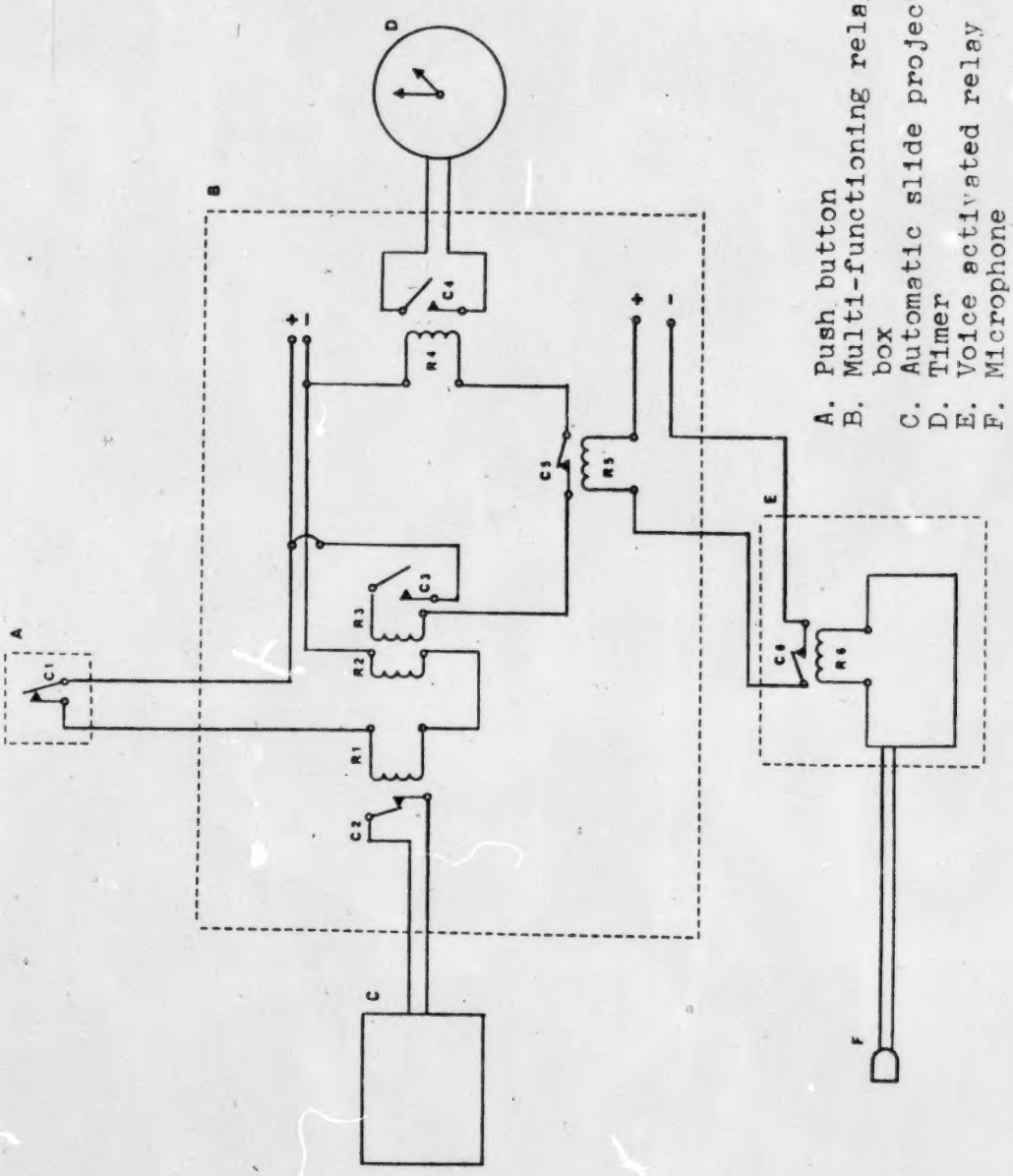
SUBJECT NO=A002 CORRECTED TIMES IN SECONDS

| PROBLEM | TRIAL= 1 | 2 | 3 | 4 | 5 | 6 | MEDIAN | MEAN | STDEV | ERRORS |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 + 0 | 1.19 | 0.98 | 0.94 | 0.89 | 0.00 | 0.00 | 0.960 | 1.000 | 0.131 | 0 |
| 0 + 1 | 0.71 | 0.74 | 0.73 | 0.70 | 0.00 | 0.00 | 0.720 | 0.720 | 0.018 | 0 |
| 0 + 2 | 0.82 | 0.83 | 0.88 | 0.67 | 0.00 | 0.00 | 0.825 | 0.800 | 0.090 | 0 |
| 0 + 3 | 0.73 | 0.81 | 0.82 | 0.89 | 0.00 | 0.00 | 0.815 | 0.812 | 0.065 | 0 |
| 0 + 4 | 0.63 | 0.63 | 0.66 | 0.74 | 0.00 | 0.00 | 0.645 | 0.665 | 0.051 | 0 |
| 0 + 5 | 0.69 | 0.67 | 0.82 | 0.77 | 0.00 | 0.00 | 0.730 | 0.737 | 0.069 | 0 |
| 0 + 6 | 0.75 | 0.81 | 1.00 | 0.76 | 0.00 | 0.00 | 0.785 | 0.830 | 0.116 | 0 |
| 0 + 7 | 0.89 | 0.58 | 0.79 | 0.80 | 0.00 | 0.00 | 0.795 | 0.765 | 0.131 | 0 |
| 0 + 8 | 0.63 | 1.14 | 0.67 | 0.71 | 0.00 | 0.00 | 0.690 | 0.787 | 0.237 | 0 |
| 0 + 9 | 0.75 | 0.75 | 0.85 | 0.64 | 0.00 | 0.00 | 0.750 | 0.747 | 0.085 | 0 |
| 0 +10 | 1.01 | 0.86 | 0.85 | 0.78 | 0.00 | 0.00 | 0.855 | 0.875 | 0.096 | 0 |
| 1 + 0 | 0.71 | 0.86 | 0.75 | 0.66 | 0.00 | 0.00 | 0.730 | 0.745 | 0.085 | 0 |
| 1 + 1 | 0.91 | 0.81 | 0.81 | 0.97 | 0.00 | 0.00 | 0.860 | 0.875 | 0.078 | 0 |
| 1 + 2 | 0.96 | 1.07 | 1.01 | 0.93 | 0.00 | 0.00 | 0.985 | 0.992 | 0.061 | 0 |
| 1 + 3 | 0.88 | 0.83 | 0.96 | 0.70 | 0.00 | 0.00 | 0.855 | 0.842 | 0.109 | 0 |
| 1 + 4 | 0.87 | 0.90 | 0.84 | 0.75 | 0.00 | 0.00 | 0.855 | 0.840 | 0.084 | 0 |
| 1 + 5 | 1.14 | 0.85 | 1.04 | 0.93 | 0.00 | 0.00 | 0.985 | 0.990 | 0.126 | 0 |
| 1 + 6 | 0.70 | 0.99 | 1.08 | 0.97 | 0.00 | 0.00 | 0.980 | 0.935 | 0.163 | 0 |
| 1 + 7 | 0.38 | 0.70 | 1.05 | 1.00 | 0.00 | 0.00 | 0.850 | 0.782 | 0.309 | 0 |
| 1 + 8 | 0.81 | 0.81 | 0.87 | 1.31 | 0.00 | 0.00 | 0.840 | 0.950 | 0.241 | 0 |
| 1 + 9 | 0.91 | 0.88 | 0.98 | 0.94 | 0.00 | 0.00 | 0.925 | 0.927 | 0.042 | 0 |
| 2 + 0 | 0.85 | 0.87 | 0.80 | 0.89 | 0.00 | 0.00 | 0.860 | 0.852 | 0.038 | 0 |
| 2 + 1 | 1.00 | 0.94 | 0.80 | 0.90 | 0.00 | 0.00 | 0.920 | 0.910 | 0.084 | 0 |
| 2 + 2 | 0.79 | 0.83 | 0.86 | 0.78 | 0.00 | 0.00 | 0.810 | 0.815 | 0.036 | 0 |
| 2 + 3 | 1.10 | ***** | 0.90 | 1.06 | 0.00 | 0.00 | 1.060 | 1.020 | 0.105 | 1 |
| 2 + 4 | 1.00 | 1.00 | 0.75 | 1.05 | 0.00 | 0.00 | 1.000 | 0.950 | 0.135 | 0 |
| 2 + 5 | 0.81 | 1.09 | 0.99 | 0.93 | 0.00 | 0.00 | 0.960 | 0.955 | 0.117 | 0 |
| 2 + 6 | 0.90 | 1.23 | 0.98 | ***** | 0.00 | 0.00 | 0.980 | 1.036 | 0.172 | 1 |
| 2 + 7 | 0.92 | ***** | 0.94 | 0.95 | 0.00 | 0.00 | 0.940 | 0.936 | 0.015 | 1 |
| 2 + 8 | 0.84 | 0.90 | 0.96 | 0.83 | 0.00 | 0.00 | 0.870 | 0.882 | 0.060 | 0 |
| 3 + 0 | 0.79 | 0.81 | 0.67 | 0.83 | 0.00 | 0.00 | 0.800 | 0.775 | 0.071 | 0 |
| 3 + 1 | 0.76 | 1.03 | 1.15 | 0.89 | 0.00 | 0.00 | 0.960 | 0.957 | 0.169 | 0 |
| 3 + 2 | 0.93 | 1.01 | 0.83 | 1.07 | 0.00 | 0.00 | 0.970 | 0.960 | 0.103 | 0 |
| 3 + 3 | 0.89 | 0.75 | 0.83 | 0.75 | 0.00 | 0.00 | 0.790 | 0.805 | 0.068 | 0 |
| 3 + 4 | 1.00 | 1.03 | 0.74 | 0.80 | 0.00 | 0.00 | 0.900 | 0.892 | 0.144 | 0 |
| 3 + 5 | 0.91 | 1.17 | 0.95 | 0.74 | 0.00 | 0.00 | 0.930 | 0.942 | 0.176 | 0 |
| 3 + 6 | 1.13 | 0.99 | 1.02 | 1.32 | 0.00 | 0.00 | 1.075 | 1.115 | 0.149 | 0 |
| 3 + 7 | 0.95 | 0.79 | 0.80 | 0.81 | 0.00 | 0.00 | 0.805 | 0.837 | 0.075 | 0 |
| 4 + 0 | 0.60 | 0.69 | 0.74 | 0.71 | 0.00 | 0.00 | 0.700 | 0.685 | 0.060 | 0 |
| 4 + 1 | 0.96 | 0.94 | 0.81 | 0.88 | 0.00 | 0.00 | 0.910 | 0.897 | 0.067 | 0 |
| 4 + 2 | 0.77 | 0.98 | 1.09 | 1.10 | 0.00 | 0.00 | 1.035 | 0.985 | 0.153 | 0 |
| 4 + 3 | 0.87 | 0.75 | 0.90 | 0.82 | 0.00 | 0.00 | 0.845 | 0.835 | 0.065 | 0 |
| 4 + 4 | 0.94 | 1.00 | 1.00 | 0.82 | 0.00 | 0.00 | 0.970 | 0.940 | 0.084 | 0 |
| 4 + 5 | 1.24 | 0.91 | 0.80 | 0.80 | 0.00 | 0.00 | 0.855 | 0.937 | 0.208 | 0 |
| 4 + 6 | 0.81 | 0.94 | 0.93 | 0.81 | 0.00 | 0.00 | 0.870 | 0.872 | 0.072 | 0 |
| 5 + 0 | 0.65 | 0.72 | 0.90 | 0.87 | 0.00 | 0.00 | 0.795 | 0.785 | 0.119 | 0 |
| 5 + 1 | 0.89 | 0.92 | 0.80 | ***** | 0.00 | 0.00 | 0.890 | 0.870 | 0.062 | 1 |
| 5 + 2 | 1.12 | 0.99 | 1.00 | 0.85 | 0.00 | 0.00 | 0.995 | 0.990 | 0.110 | 0 |
| 5 + 3 | 1.04 | 0.92 | 0.85 | 2.38 | 0.00 | 0.00 | 0.980 | 1.297 | 0.725 | 0 |
| 5 + 4 | 0.93 | 1.01 | 0.87 | 0.82 | 0.00 | 0.00 | 0.900 | 0.907 | 0.081 | 0 |
| 5 + 5 | 0.86 | 0.93 | 0.78 | 0.77 | 0.00 | 0.00 | 0.820 | 0.835 | 0.075 | 0 |
| 6 + 0 | 0.67 | 0.76 | 0.86 | 0.82 | 0.00 | 0.00 | 0.790 | 0.777 | 0.082 | 0 |
| 6 + 1 | 0.82 | 0.81 | 0.83 | 0.68 | 0.00 | 0.00 | 0.815 | 0.785 | 0.070 | 0 |
| 6 + 2 | 1.12 | 1.14 | 1.17 | 0.91 | 0.00 | 0.00 | 1.130 | 1.085 | 0.118 | 0 |
| 6 + 3 | 1.05 | ***** | 1.04 | 0.82 | 0.00 | 0.00 | 1.040 | 0.970 | 0.130 | 1 |
| 6 + 4 | ***** | 0.85 | 0.85 | 0.91 | 0.00 | 0.00 | 0.850 | 0.870 | 0.034 | 1 |
| 7 + 0 | 0.78 | 0.73 | 0.71 | 0.82 | 0.00 | 0.00 | 0.755 | 0.760 | 0.049 | 0 |
| 7 + 1 | 1.00 | 0.72 | 1.03 | 0.85 | 0.00 | 0.00 | 0.925 | 0.900 | 0.143 | 0 |
| 7 + 2 | 1.07 | ***** | 0.82 | 0.89 | 0.00 | 0.00 | 0.890 | 0.926 | 0.128 | 1 |
| 7 + 3 | 0.98 | 1.00 | 0.93 | 1.04 | 0.00 | 0.00 | 0.990 | 0.987 | 0.045 | 0 |
| 8 + 0 | 0.70 | 0.86 | 0.73 | 0.74 | 0.00 | 0.00 | 0.735 | 0.757 | 0.070 | 0 |
| 8 + 1 | 0.92 | 1.05 | 0.79 | 0.85 | 0.00 | 0.00 | 0.885 | 0.902 | 0.111 | 0 |
| 8 + 2 | 0.97 | 0.79 | 0.80 | 0.76 | 0.00 | 0.00 | 0.795 | 0.830 | 0.094 | 0 |
| 9 + 0 | 0.70 | 0.73 | 0.93 | 0.83 | 0.00 | 0.00 | 0.780 | 0.797 | 0.104 | 0 |
| 9 + 1 | 0.81 | 1.03 | ***** | 0.89 | 0.00 | 0.00 | 0.890 | 0.910 | 0.111 | 1 |
| 10 + 0 | 0.83 | 0.98 | 0.83 | 0.86 | 0.00 | 0.00 | 0.845 | 0.875 | 0.071 | 0 |
| ERRORS | 1 | 4 | 1 | 2 | 0 | 0 | | | TOTAL | 8 |

Exhibit 3.

Appendix C

Wiring diagram of the equipment used and description of the test-cycle



Appendix C (con'd)

Description of the test-cycle

1. E starts the test cycle by pressing pushbutton (A) which closes contact C1. This closes the circuit in the multi-functioning auxiliary relay-box (B) containing relays R1 and R2 which are then activated.

When R1 is activated, C2 is closed and the remote-controlled slide projector (C) begins its slide-changing cycle. C5 is closed in its normal unloaded state. Thus, when R2 is activated which causes C3 to close, the circuit containing relays R3 and R4 is closed and R3 and R4 are activated. R3 keeps C3 and the circuit C3-R3-C5-R4 closed even after R2 is deactivated by E's releasing pushbutton (A) and opening C1.

When R4 is activated it closes C4, the timer circuit is closed and the timing of the response latency has begun.

2. When S has formulated an answer and speaks into the microphone (F), the signals from the microphone are amplified by the voice key (E) and the circuit containing R5 is closed and thus activated. C5 opens and the circuit C3-R3-C5-R4 is opened.

R4 is deactivated, C4 opened, the timer circuit opened, and the timer (D) stops. The timer has recorded

the total time from the start of the slide-changing cycle to S's initiation of response.

C3 is also opened and can not be closed again until E presses the pushbutton (A). Thus any signals from the microphone, after the initial sound, can not reactivate the timer (D).

3. After E has recorded the response time, pressed the pushbutton (A) once to change to a blank slide, and reset the timer (D), the cycle is then started again from point 1 above.